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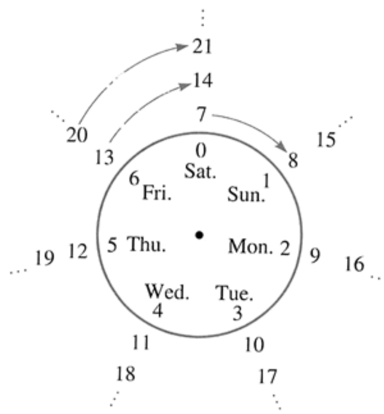
Chapter Five

Section four

Modular System

The days of the week cycle are repeatedly through the numbers 0, 1, 2, 3, 4, 5, and 6. See the following figure from page 234.



If today is Thursday, July 25, 2002, what day of the week will it be one year from today?

$365 \div 7 = 52$, Remainder = 1. This means we would be going around the 7-day clock 52 times plus 1 more days. One day past Thursday is Friday, so one year from today will be a Friday.

If today is Thursday, July 25, 2002, what day of the week will it be September 4, 2002?

$31 - 25 = 6,\quad 6 + 31 + 4 = 41,\quad 41 \div 7 = 5,\quad R = 6,\quad 41 \equiv 6 \equiv -1 \pmod 7$

That is, one day before Thursday or 6 days after Thursday is Wednesday.

If today is Thursday, July 25, 2002, what day of the week was it April 26, 2002?

$30 - 26 = 4,\quad 4 + 31 + 30 + 25 = 90,\quad -90 \div 7 = -12,\quad R = -6,\quad -6 + 7 = 1,\quad -90 \equiv -6 \equiv 1 \pmod 7$

It is 1 day after Thursday. That is, Friday.

Definition:

$a \equiv b \pmod m$ if and only if $a - b = km$ for some integer k.

$a \equiv b \pmod m$ if and only if the same remainder is obtained when a and b are divided by m.

Examples:

Decide whether each statement is *true* or *false*.

1. $16 \equiv 10 \pmod 2$ T (why?)
2. $49 \equiv 32 \pmod 7$ F
3. $30 \equiv 345 \pmod 7$ T

Find the modulus value of 25 (mod 7). $25 \div 7 = 3$ (R=4), so, $25 \equiv 4 \pmod 7$

Find the modulus value of $-17 \pmod 7$.

$-17 \div 7 = -2$ (R$=-3$), $-3+7=4$, so $-17 \equiv 4 \pmod 7$

Find each of the following sums, differences, and products.

1. (9+14) (mod 3) 2
2. (27−5) (mod 6) 4
3. (50+34) (mod 7) 0
4. $(8 \times 9) \pmod{10}$ 2
5. $(12 \times 10) \pmod 5$ 0
6. $(6 - 24) \pmod 4$ 2

A laboratory experiment is begun at 9 o'clock. If the experiment lasts for 43 hours, at what time of day will it end?

We look for $(9+43) \pmod{24}$.
$52 \div 24 = 2$ R $= 4$. That is, $(9+43) \equiv 4 \pmod{24}$.
Thus, the experiment will end at 4 o'clock.

Exercises:

Solve each problem using modular arithmetic.

5. $(12 + 7)\pmod 4$
6. $(62 + 95)\pmod 9$
7. $(35 - 22)\pmod 5$
8. $(82 - 45)\pmod 3$
9. $(5 \times 8)\pmod 3$
10. $(32 \times 21)\pmod 8$
11. $[4 \times (13 + 6)]\pmod{11}$
12. $[(10 + 7) \times (5 + 3)]\pmod{10}$
13. $(3 - 27)\pmod 5$
14. $(16 - 60)\pmod 7$
15. $[(-8) \times 11]\pmod 3$
16. $[2 \times (-23)]\pmod 5$

Answers:

5. 3 6. 4 7. 3 8. 1 9. 1 10. 0 11. 10
12. 6 13. 1 14. 5 15. 2 16. 4