

Chapter Five
Section one

- One counting number is **divisible** by another if the operation of dividing the first number by the second leaves a *remainder 0*. If y divides x , then we write $y \mid x$.
- If the natural number x is divisible by the natural number y , then y is a **factor** (or **divisor**) of x , and x is a **multiple** of y .
- A natural number greater than 1 that *has only itself and 1 as factors* is called a **prime** number.
- A natural number greater than 1 that is not a prime number is called a **composite** number.
- 1 is neither a prime number nor a composite number.
- If a natural number is written as a product of several its prime factors, we call the process a **prime factorization** of a number or **completely factor** a number.

Exercises:

State whether each of the following numbers is *prime* or *composite*.

9	5	17	27	31
2	15	47	53	67
81	87	93	73	49
59	79	69	23	63
121	97	101	89	105
207	145	113	697	276

Find the *prime factorization* of each of the following numbers. (*Completely factor* each of the following numbers.)

90	84	88	54	42
63	32	45	108	144
64	169	210	425	950

- The following shows 25 prime numbers from 2 to 100. (Page 211)
How do you create it? (Think!!!)

TABLE 1 Sieve of Eratosthenes

	②	③	4	⑤	6	⑦	8	9	10	⑪	12	⑬	14
15	16	⑰	18	⑱	20	21	22	⑳	24	25	26	27	28
⑳	30	⑳	32	33	34	35	36	㉑	38	39	40	㉒	42
④③	44	45	46	④⑦	48	49	50	51	52	⑤③	54	55	56
57	58	⑤⑨	60	⑥①	62	63	64	65	66	⑥⑦	68	69	70
⑦①	72	⑦③	74	75	76	77	78	⑦⑨	80	81	82	⑧③	84
85	86	87	88	⑧⑨	90	91	92	93	94	95	96	⑨⑦	98
99	100												

- The following shows a list of divisibility tests. (Page 212)

TABLE 2 Divisibility Tests		
Divisible By	Test	Example
2	Number ends in 0, 2, 4, 6, or 8. (The last digit is even.)	9,489,994 ends in 4; it is divisible by 2.
3	Sum of the digits is divisible by 3.	897,432 is divisible by 3, since $8 + 9 + 7 + 4 + 3 + 2 = 33$ is divisible by 3.
4	Last two digits form a number divisible by 4.	7,693,432 is divisible by 4, since 32 is divisible by 4.
5	Number ends in 0 or 5.	890 and 7635 are divisible by 5.
6	Number is divisible by both 2 and 3.	27,342 is divisible by 6 since it is divisible by both 2 and 3.
8	Last three digits form a number divisible by 8.	1,437,816 is divisible by 8, since 816 is divisible by 8.
9	Sum of the digits is divisible by 9.	428,376,105 is divisible by 9 since sum of digits is 36, which is divisible by 9.
10	The last digit is 0.	897,463,940 is divisible by 10.
12	Number is divisible by both 4 and 3.	376,984,032 is divisible by 12.

Page 218:

Here is a divisibility test for 7.

- Double the last digit of the given number, and subtract this value from the given number with the last digit omitted.*
- Repeat the process of part (a) as many times as necessary until the number obtained can easily be divided by 7.*
- If the final number obtained is divisible by 7, then the given number also is divisible by 7. If the final number is not divisible by 7, then neither is the given number.*

Use this divisibility test to determine whether each of the following is divisible by 7.

- 37.** 142,891 **38.** 409,311 **39.** 458,485 **40.** 287,824

Here is a divisibility test for 11.

- Starting at the left of the given number, add together every other digit.*
- Add together the remaining digits.*
- Subtract the smaller of the two sums from the larger. (If they are the same, the difference is 0.)*
- If the final number obtained is divisible by 11, then the given number also is divisible by 11. If the final number is not divisible by 11, then neither is the given number.*

Use this divisibility test to determine whether each of the following is divisible by 11.

- 41.** 8,493,969 **42.** 847,667,942 **43.** 453,896,248 **44.** 552,749,913

Exercises:

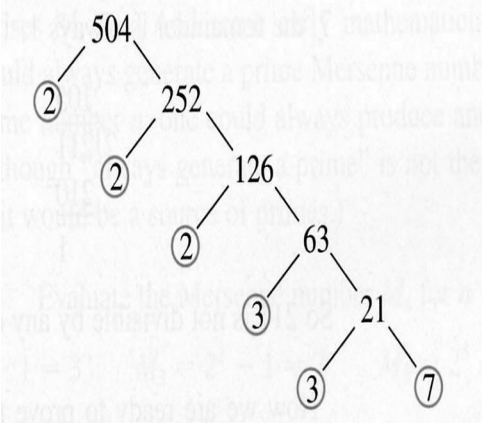
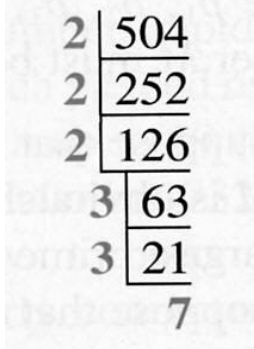
Is 2,984,094 divisible by 4? (No, why?)

Is 4,119,806,514 divisible by 9? (No, how to check?)

Is 315 divisible by 3?

Is 123,456,789 divisible by 3 or 4?

Find the prime factorization of 504.

Factor Tree	Compact Form
 <p>A factor tree for 504. The root is 504. It branches into 2 and 252. 252 branches into 2 and 126. 126 branches into 2 and 63. 63 branches into 3 and 21. 21 branches into 3 and 7. All prime factors are circled.</p> $504 = 2^3 \cdot 3^2 \cdot 7$	 <p>A compact form showing the division of 504 by its prime factors: 2, 2, 2, 3, 3, and 7.</p> $504 = 2^3 \cdot 3^2 \cdot 7$

The Fundamental Theorem of Arithmetic: Every composite natural number can be expressed in one and only one way as a product of primes (if the order of the factors is disregarded).

There is no largest prime number.