

**Chapter Four**  
Section three

Conversion Between Number Bases

- **Binary Numbers: (Base Two)**

ON	OFF
1	0

Convert Binary to Decimal:

$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	Decimals
32	16	8	4	2	1	
off	on 1	off 0	off 0	on 1	off 0	$1 \times 16 + 1 \times 2 = 18$
off	on 1	on 1	off 0	on 1	on 1	$1 \times 16 + 1 \times 8 + 1 \times 2 + 1 \times 1 = 27$
on 1	on 1	on 1	on 1	off 0	off 0	$1 \times 32 + 1 \times 16 + 1 \times 8 + 1 \times 4 = 60$

Convert Decimal to Binary:

1. Divide the decimal number by two and record remainder.
2. Divide quotient by two and record remainder.
3. Keep dividing quotients and recording remainders until getting 0 as quotient.
4. Write down all remainders by the backward method.

Example:

$$\begin{array}{r}
 34 \div 2 = 17, \quad R=0 \\
 17 \div 2 = 8, \quad R=1 \quad \uparrow \\
 8 \div 2 = 4, \quad R=0 \\
 4 \div 2 = 2, \quad R=0 \\
 2 \div 2 = 1, \quad R=0 \\
 1 \div 2 = 0, \quad R=1
 \end{array}$$

So,  $34_{ten} = 100010_{two}$

- **Convert from another base to decimal form.**

Calculator Shortcut

$$\begin{aligned}
 1342_{five} &= 1 \times 5^3 + 3 \times 5^2 + 4 \times 5^1 + 2 \times 5^0 \\
 &= 1 \times 125 + 3 \times 25 + 4 \times 5 + 2 \times 1 \\
 &= 222
 \end{aligned}$$

$$\begin{aligned}
 &= (1 \times 5^2 + 3 \times 5 + 4) \times 5 + 2 \\
 &= [(1 \times 5 + 3) \times 5 + 4] \times 5 + 2 \\
 &= 222
 \end{aligned}$$

$$\begin{aligned}
 244314_{five} &= (((((2 \times 5 + 4) \times 5 + 4) \times 5 + 3) \times 5 + 1) \times 5 + 4 \\
 &= (((14 \times 5 + 4) \times 5 + 3) \times 5 + 1) \times 5 + 4 \\
 &= ((74 \times 5 + 3) \times 5 + 1) \times 5 + 4 \\
 &= (373 \times 5 + 1) \times 5 + 4 \\
 &= 1866 \times 5 + 4 = 9334
 \end{aligned}$$

6 digits start with 4 ‘(‘.

$$6343_{\text{seven}} = ((6 \times 7 + 3) \times 7 + 4) \times 7 + 3 = (45 \times 7 + 4) \times 7 + 3 = 319 \times 7 + 3 = 2236$$

$$110101_{\text{two}} = (((1 \times 2 + 1) \times 2 + 0) \times 2 + 1) \times 2 + 0) \times 2 + 1 = 53$$

- **Convert from decimal form to another base.**

Convert 497 from decimal form to base five.

$497 \div 5 = 99$	R=2	↑
$99 \div 5 = 19$	R=4	
$19 \div 5 = 3$	R=4	
$3 \div 5 = 0$	R=3	

$$497 = 3442_{\text{five}}$$

Convert 7508 from decimal form to base seven.

$7508 \div 7 = 1072$	R=4	↑
$1072 \div 7 = 153$	R=1	
$153 \div 7 = 21$	R=6	
$21 \div 7 = 3$	R=0	
$3 \div 7 = 0$	R=3	

$$7508 = 30614_{\text{seven}}$$

- **Convert from one base other than decimal form to another base.**

Convert  $3164_{\text{seven}}$  to base five.

- Convert from another base to decimal form first.
- Then, convert from decimal form to the other base.

$$\begin{aligned} 3164_{\text{seven}} &= ((3 \times 7 + 1) \times 7 + 6) \times 7 + 4 \\ &= (22 \times 7 + 6) \times 7 + 4 \\ &= 160 \times 7 + 4 \\ &= 1124 \end{aligned}$$

$1124 \div 5 = 224$	R=4	↑
$224 \div 5 = 44$	R=4	
$44 \div 5 = 8$	R=4	
$8 \div 5 = 1$	R=3	
$1 \div 5 = 0$	R=1	

$$3164_{\text{seven}} = 1124 = 13444_{\text{five}}$$

- **Base sixteen** requires more symbols than are normally used in our decimal system. Computer programmers commonly use the letters **A, B, C, D, E, and F** as hexadecimal digits for the numbers the through fifteen, respectively.

Convert  $FA5_{\text{sixteen}}$  to decimal form.

$$FA5_{\text{sixteen}} = (15 \times 16 + 10) \times 16 + 5 = 250 \times 16 + 5 = 4005$$

Convert 748 from decimal form to hexadecimal form.

$748 \div 16 = 46$	R=12=C	↑
$46 \div 16 = 2$	R=14=E	
$2 \div 16 = 0$	R=2	

$$748 = 2EC_{\text{sixteen}}$$

- Common Computer-Oriented Bases:

**TABLE 6** Some Decimal Equivalents in the Common Computer-Oriented Bases

Decimal (Base Ten)	Hexadecimal (Base Sixteen)	Octal (Base Eight)	Binary (Base Two)
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
10	A	12	1010
11	B	13	1011
12	C	14	1100
13	D	15	1101
14	E	16	1110
15	F	17	1111
16	10	20	10000
17	11	21	10001

**Exercises:**

Convert each of the following to decimal form by expanding in powers and by using the calculator shortcut.

- |                            |                             |                           |                              |
|----------------------------|-----------------------------|---------------------------|------------------------------|
| 15. $24_{\text{five}}$     | 16. $62_{\text{seven}}$     | 17. $1011_{\text{two}}$   | 18. $35_{\text{eight}}$      |
| 19. $3BC_{\text{sixteen}}$ | 20. $34432_{\text{five}}$   | 21. $2366_{\text{seven}}$ | 22. $101101110_{\text{two}}$ |
| 23. $70266_{\text{eight}}$ | 24. $ABCD_{\text{sixteen}}$ | 25. $2023_{\text{four}}$  | 26. $6185_{\text{nine}}$     |
| 27. $41533_{\text{six}}$   |                             | 28. $88703_{\text{nine}}$ |                              |

Convert each of the following from decimal form to the given base.

- |                         |                         |                          |
|-------------------------|-------------------------|--------------------------|
| 29. 86 to base five     | 30. 65 to base seven    | 31. 19 to base two       |
| 32. 935 to base eight   | 33. 147 to base sixteen | 34. 2730 to base sixteen |
| 35. 36401 to base five  | 36. 70893 to base seven | 37. 586 to base two      |
| 38. 12888 to base eight | 39. 8407 to base three  | 40. 11028 to base four   |
| 41. 9346 to base six    | 42. 99999 to base nine  |                          |

**Solutions:**

- |                             |                               |                            |                                |                             |                            |                             |            |
|-----------------------------|-------------------------------|----------------------------|--------------------------------|-----------------------------|----------------------------|-----------------------------|------------|
| 21. 881                     | 22. 366                       | 23. 28,854                 | 24. 43,981                     | 25. 139                     | 26. 4532                   | 27. 5601                    | 28. 58,890 |
| 29. $321_{\text{five}}$     | 30. $122_{\text{seven}}$      | 31. $10011_{\text{two}}$   | 32. $1647_{\text{eight}}$      | 33. $93_{\text{sixteen}}$   | 34. $AAA_{\text{sixteen}}$ | 35. $2131101_{\text{five}}$ |            |
| 36. $413454_{\text{seven}}$ | 37. $1001001010_{\text{two}}$ | 38. $31130_{\text{eight}}$ | 39. $102112101_{\text{three}}$ | 40. $2230110_{\text{four}}$ | 41. $111134_{\text{six}}$  |                             |            |
| 42. $162150_{\text{nine}}$  |                               |                            |                                |                             |                            |                             |            |