Chapter Three

Sections three, four

Conditionals:

Truth Table for the Conditional If p, then q

If p, then q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is the antecedent.
q is the consequent.

$p \to q$ p implies q.
p is a sufficient condition (hypothesis)
q is a necessary condition (conclusion)

- $p \to q$ is false only when the antecedent is true and the consequent is false.
- If the antecedent is false, then $p \to q$ is automatically true.
- If consequent is true, then $p \to q$ is automatically true.

Example:

If I read for too long, then I get a headache.

The above statement will be false only if I read for too long and I do not get a headache.
I may get a headache because of some other reasons. It is not necessary to be reading too long.
If I do not read too long and I do not get a headache, it is nothing wrong with the above statement.

If I pass Mathematics, then the sun will rise the next day.

The sun will rise no matter what grade I get in a course.

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

So, the statement $\sim p \vee q$ is equivalent to $p \to q$.

Examples:

Construct the truth table for $p \to \sim(p \wedge q)$.

SOLUTION

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \to \sim(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Construct the truth table for $p \vee \sim q \rightarrow \sim p \wedge q$.

SOLUTION

p	q	$\sim q$	$p \vee \sim q$	$\sim p$	q	$\sim p \wedge q$	$p \vee \sim q \rightarrow \sim p \wedge q$
T	T	F	T	F	T	F	F
T	F	T	T	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	T	T	F	F	F

Definitions:

Related Conditional Statements

Direct Statement	$p \rightarrow q$	(If p, then q.)
Converse	$q \rightarrow p$	(If q, then p.)
Inverse	$\sim p \rightarrow \sim q$	(If not p, then not q.)
Contrapositive	$\sim q \rightarrow \sim p$	(If not q, then not p.)

Equivalent

Equivalent

		Direct	Converse	Inverse	Contrapositive
p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Biconditionals The compound statement p *if and only if* q (often abbreviated p *iff* q) is called a **biconditional**. It is symbolized $p \leftrightarrow q$, and is interpreted as the conjunction of the two conditionals $p \rightarrow q$ and $q \rightarrow p$. Using symbols, this conjunction is written

$$(q \rightarrow p) \wedge (p \rightarrow q)$$

so that, by definition,

$$p \leftrightarrow q \equiv (q \rightarrow p) \wedge (p \rightarrow q).$$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercises:

Let b *represent* "I ride my bike," *let* r *represent* "it rains," *and let* p *represent* "the play is cancelled." *Write each compound statement in symbols.*

33. If it rains, then I ride my bike.

34. If I ride my bike, then the play is cancelled.

35. If I do not ride my bike, then it does not rain.

36. If the play is cancelled, then it does not rain.

37. I ride my bike, or if the play is cancelled then it rains.

38. The play is cancelled, and if it rains then I do not ride my bike.

39. I'll ride my bike if it doesn't rain.

40. It rains if the play is cancelled.

Find the truth value of each statement. Assume that p *and* r *are false, and* q *is true.*

41. $\sim r \rightarrow q$

42. $\sim p \rightarrow \sim r$

43. $q \rightarrow p$

44. $\sim r \rightarrow p$

45. $p \rightarrow q$

46. $\sim q \rightarrow r$

Solutions:

33. $r \rightarrow b$

34. $b \rightarrow p$ **35.** $\sim b \rightarrow \sim r$ **36.** $p \rightarrow \sim r$ **37.** $b \vee (p \rightarrow r)$ **38.** $p \wedge (r \rightarrow \sim b)$ **39.** $\sim r \rightarrow b$

40. $p \rightarrow r$ **41.** T **42.** T **43.** F **44.** F **45.** T **46.** T