

**Chapter Two**  
Sections three & four

Basic operations in real numbers:  $+$ ,  $-$ ,  $\times$ ,  $\div$ .

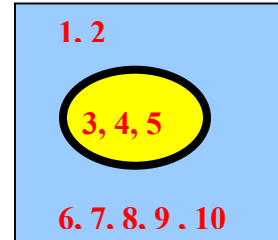
Common operations in sets:  $\cap$ ,  $\cup$ ,  $A'$ ,  $-$ ,  $\times$ .

1. **Complement** of  $A$ , written  $A'$ , is  $A' = \{x \mid x \in U \text{ and } x \notin A\}$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{3, 4, 5\}$$

$$A' = \{1, 2, 6, 7, 8, 9, 10\}$$



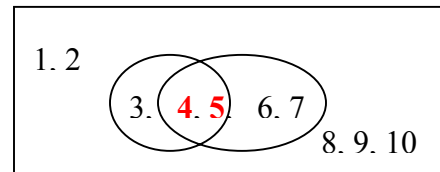
2. **Intersection** of  $A$  and  $B$  is  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{3, 4, 5\} \text{ and } B = \{4, 5, 6, 7\}$$

$$A \cap B = \{4, 5\} \quad A \cap \phi = \phi \text{ (Why?)}$$

$$A \cap A' = \phi \quad A' \cap B = ? \text{ (Think!)}$$



3. **Union** of  $A$  and  $B$  is  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

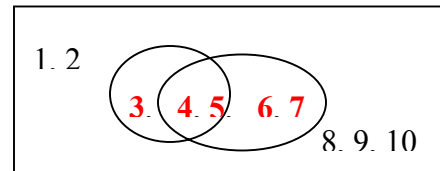
$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{3, 4, 5\} \text{ and } B = \{4, 5, 6, 7\}$$

$$A \cup B = \{3, 4, 5, 6, 7\}$$

$$A \cup A' = U \quad A \cup \phi = A \text{ (Why?)}$$

$$A' \cup B' = ? \text{ (Think!)}$$



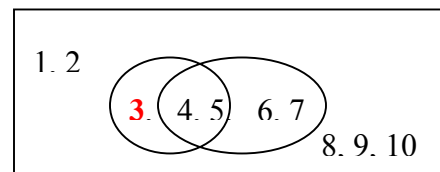
4. **Difference** of  $A$  and  $B$  is  $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{3, 4, 5\} \text{ and } B = \{4, 5, 6, 7\}$$

$$A - B = \{3\}$$

Is  $A - B = B - A$ ?



**Fact:**  $A - B = A \cap B'$

5. **Cartesian Product** of  $A$  and  $B$  is  $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

$$A = \{1, 2\} \text{ and } B = \{3, 4, 5\}$$

$$A \times B = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

**Notes:**  $n(A \times B) = n(A) \times n(B)$  where  $n(A)$  is the number of elements in  $A$ .

**Example:**

$A = \{x, y, z, u, v, w\}$ ,  $B = \{a, e, i, o, u\}$ , and  $C = \{x, y, z, a, u\}$ .

Draw Venn Diagrams to find the following solutions.

(a)  $(A \cap B) \cup C$

(b)  $A \cap (B \cup C)$

(c) Is  $(A \cap B) \cup C = A \cap (B \cup C)$ ?

**Define the order of operation: left to right**

$$A \cap B \cup C = (A \cap B) \cup C$$

$$A \cup B \cap C = (A \cup B) \cap C$$

**De Morgan's Laws**

For any sets  $A$  and  $B$ ,  $(A \cap B)' = A' \cup B'$  and  $(A \cup B)' = A' \cap B'$

**Note:** Use Venn Diagrams and the following example to De Morgan's Laws

$U = \{a, e, i, o, u\}$ ,  $A = \{e, i\}$  and  $B = \{a, e, u\}$

Does **Commutative Law** hold? (Can they change order?)

$$A \cup B = B \cup A, \quad A \cap B = B \cap A \quad \text{but} \quad A - B \neq B - A$$

Does **Associative Law** hold?

$$(A \cup B) \cup C = A \cup (B \cup C) \quad \text{and} \quad (A \cap B) \cap C = A \cap (B \cap C)$$

Does **Distributive Law** hold?

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad \text{and} \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

**Fact:**

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If  $(A \cap B) = \phi$  ( $A$  and  $B$  are disjoint) then  $n(A \cup B) = n(A) + n(B)$

Translate English to Algebra:

|         |        |        |                   |
|---------|--------|--------|-------------------|
| English | AND    | OR     | NOT A             |
| Algebra | $\cap$ | $\cup$ | $A'$ (Complement) |

**Example:** Survey in the class

Who likes apple?

Who likes banana?

Who likes both?

Who doesn't like any of them?

|        |     | Apple |    | Total |
|--------|-----|-------|----|-------|
|        |     | Yes   | No |       |
| Banana | Yes |       |    |       |
|        | No  |       |    |       |
| Total  |     |       |    |       |

Find  $n(A)$ ,  $n(B)$ ,  $n(A \cap B)$ , and  $n(A \cup B)$ .

Examples:

- Find  $n(A)$  if  $n(A \cup B) = 22$ ,  $n(A \cap B) = 8$  and  $n(B) = 12$
- Find  $n(A \cup B)$  if  $n(A) = 10$ ,  $n(B) = 6$  and  $n(A \cap B) = 4$
- Find  $n(A \cap B)$  if  $n(A) = 20$ ,  $n(B) = 30$  and  $n(A \cup B) = 43$
- Find  $n(A \cup B)$  if  $n(A) = 4$ ,  $n(B) = 5$  and  $A \cap B = \phi$ .

Exercises: (Section 3)

*Perform the indicated operations.*

Let  $U = \{a, b, c, d, e, f, g\}$   
 $X = \{a, c, e, g\}$   
 $Y = \{a, b, c\}$   
 $Z = \{b, c, d, e, f\}$ .

7.  $X \cap Y$

8.  $X \cup Y$

9.  $Y \cup Z$

10.  $Y \cap Z$

11.  $X \cup U$

12.  $Y \cap U$

13.  $X'$

14.  $Y'$

15.  $X' \cap Y'$

16.  $X' \cap Z$

17.  $X \cup (Y \cap Z)$

18.  $Y \cap (X \cup Z)$

19.  $(Y \cap Z') \cup X$

20.  $(X' \cup Y') \cup Z$

21.  $(Z \cup X')' \cap Y$

22.  $(Y \cap X')' \cup Z'$

23.  $X - Y$

24.  $Y - X$

25.  $X' - Y$

26.  $Y' - X$

27.  $X \cap (X - Y)$

28.  $Y \cup (Y - X)$

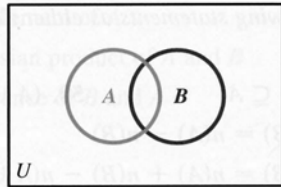
For Exercises 55–62, use your results in the earlier part(s) to answer the last part.

$$\begin{aligned} \text{Let } U &= \{1, 2, 3, 4, 5\} \\ X &= \{1, 3, 5\} \\ Y &= \{1, 2, 3\} \\ Z &= \{3, 4, 5\}. \end{aligned}$$

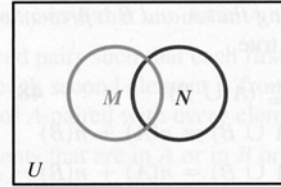
55. a) Find  $X \cup Y$ . b) Find  $Y \cup X$ . c) State a conjecture.
56. a) Find  $X \cap Y$ . b) Find  $Y \cap X$ . c) State a conjecture.
57. a) Find  $X \cup (Y \cap Z)$ . b) Find  $(X \cup Y) \cap Z$ . c) State a conjecture.
58. a) Find  $X \cap (Y \cap Z)$ . b) Find  $(X \cap Y) \cap Z$ . c) State a conjecture.
59. a) Find  $(X \cup Y)'$ . b) Find  $X' \cap Y'$ . c) State a conjecture.
60. a) Find  $(X \cap Y)'$ . b) Find  $X' \cup Y'$ . c) State a conjecture.
61. a) Find  $X \cup \emptyset$ . b) State a conjecture.
62. a) Find  $X \cap \emptyset$ . b) State a conjecture.

Place the elements of these sets in the proper locations on the given Venn diagram.

81. Let  $U = \{a, b, c, d, e, f, g\}$   
 $A = \{b, d, f, g\}$   
 $B = \{a, b, d, e, g\}$ .

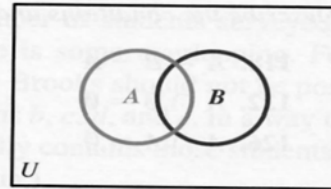


82. Let  $U = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$   
 $M = \{5, 8, 10, 11\}$   
 $N = \{5, 6, 7, 9, 10\}$ .



Use a Venn diagram similar to the one shown here to shade each of the following sets.

83.  $B \cap A'$       84.  $A \cup B$       85.  $A' \cup B$   
 86.  $A' \cap B'$       87.  $B' \cup A$       88.  $A' \cup A$   
 89.  $B' \cap B$       90.  $A \cap B'$       91.  $B' \cup (A' \cap B')$   
 92.  $(A \cap B) \cup B$       93.  $U'$       94.  $\emptyset'$

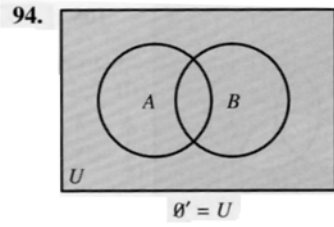
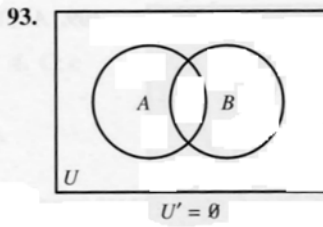
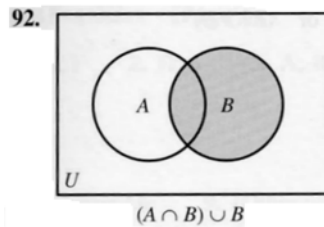
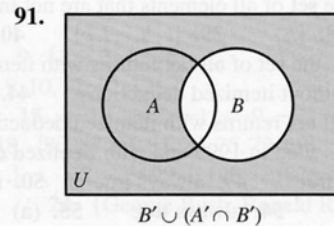
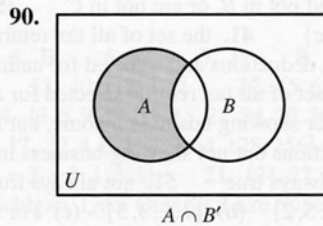
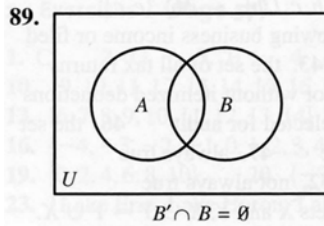
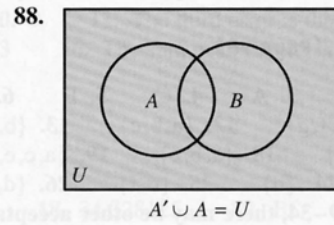
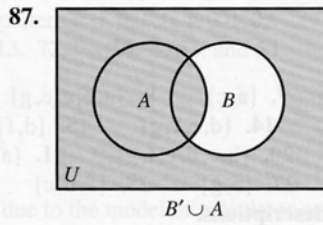
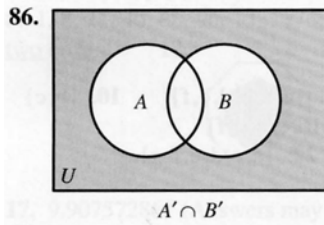
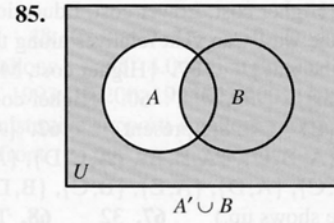
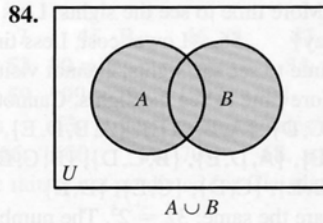
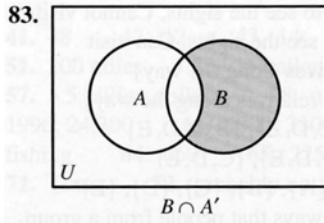
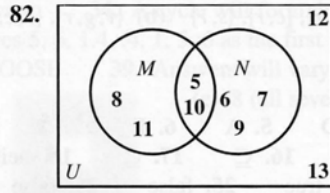
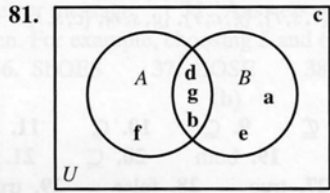


Solutions:

7.  $\{a, c\}$     8.  $\{a, b, c, e, g\}$     9.  $\{a, b, c, d, e, f\}$     10.  $\{b, c\}$   
 11.  $\{a, b, c, d, e, f, g\}$     12.  $\{a, b, c\}$     13.  $\{b, d, f\}$     14.  $\{d, e, f, g\}$     15.  $\{d, f\}$     16.  $\{b, d, f\}$   
 17.  $\{a, b, c, e, g\}$     18.  $\{a, b, c\}$     19.  $\{a, c, e, g\}$     20.  $\{b, c, d, e, f, g\}$     21.  $\{a\}$     22.  $\{a, c, d, e, f, g\}$   
 23.  $\{e, g\}$     24.  $\{b\}$     25.  $\{d, f\}$     26.  $\{d, f\}$     27.  $\{e, g\}$     28.  $\{a, b, c\}$

55. (a)  $\{1, 3, 5, 2\}$  (b)  $\{1, 2, 3, 5\}$  (c) For any sets  $X$  and  $Y$ ,  $X \cup Y = Y \cup X$ .

56. (a)  $\{1, 3\}$  (b)  $\{1, 3\}$  (c) For any sets  $X$  and  $Y$ ,  $X \cap Y = Y \cap X$ .    57. (a)  $\{1, 3, 5, 2, 4\}$  (b)  $\{1, 3, 5, 2, 4\}$   
 (c) For any sets  $X$ ,  $Y$ , and  $Z$ ,  $X \cup (Y \cap Z) = (X \cup Y) \cap Z$ .    58. (a)  $\{3\}$  (b)  $\{3\}$  (c) For any sets  $X$ ,  $Y$ , and  $Z$ ,  
 $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ .    59. (a)  $\{4\}$  (b)  $\{4\}$  (c) For any sets  $X$  and  $Y$ ,  $(X \cup Y)' = X' \cap Y'$ .  
 60. (a)  $\{2, 4, 5\}$  (b)  $\{2, 4, 5\}$  (c) For any sets  $X$  and  $Y$ ,  $(X \cap Y)' = X' \cup Y'$ .    61. (a)  $\{1, 3, 5\}$  (b) For any set  $X$ ,  
 $X \cup \emptyset = X$ .    62. (a)  $\emptyset$  (b) For any set  $X$ ,  $X \cap \emptyset = \emptyset$ .



Section 4:

To prepare for the survey problems that follow later in this exercise set, draw an appropriate Venn diagram and use the given information to fill in the number of elements in each region.

- 9.  $n(U) = 43, n(A) = 25, n(A \cap B) = 5, n(B') = 30$
- 10.  $n(A) = 19, n(B) = 13, n(A \cup B) = 25, n(A') = 11$
- 11.  $n(A \cup B) = 15, n(A \cap B) = 8, n(A) = 13, n(A' \cup B') = 11$
- 12.  $n(A') = 25, n(B) = 28, n(A' \cup B') = 40, n(A \cap B) = 10$
- 13.  $n(A) = 24, n(B) = 24, n(C) = 26, n(A \cap B) = 10, n(B \cap C) = 8, n(A \cap C) = 15, n(A \cap B \cap C) = 6, n(U) = 50$

22. *Wine Tasting* The following list shows the preferences of 102 people at a wine-tasting party:

- 99 like Spañada
- 96 like Ripple
- 99 like Boone's Farm Apple Wine
- 95 like Spañada and Ripple
- 94 like Ripple and Boone's
- 96 like Spañada and Boone's
- 93 like all three.

How many people like:

- a) none of the three?
- b) Spañada, but not Ripple?
- c) anything but Boone's Farm?
- d) only Ripple?
- e) exactly two kinds of wine?

24. *Poultry on a Farm* Old MacDonald surveyed her flock with the following results. She has:

- 9 fat red roosters      18 thin brown roosters
- 2 fat red hens        6 thin red roosters
- 26 fat roosters        5 thin red hens
- 37 fat chickens        7 thin brown hens.

Answer the following questions about the flock. [Hint: You need a Venn diagram with circles for fat, for male (a rooster is a male; a hen is a female), and for red (assume that brown and red are opposites in the chicken world).] How many chickens are:

- a) fat?
- b) red?
- c) male?
- d) fat, but not male?
- e) brown, but not fat?
- f) red and fat?

25. *Student Goals* Julie Ward, who sells college textbooks, interviewed freshmen on a community college campus to find out the main goals of today's students.

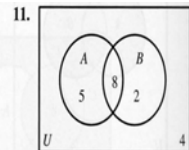
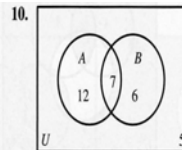
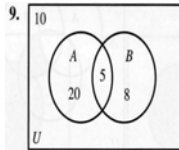
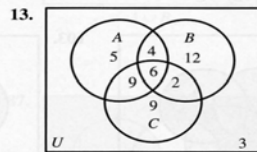
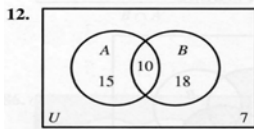
- Let  $W$  = the set of those who want to be wealthy
- $F$  = the set of those who want to raise a family
- $E$  = the set of those who want to become experts in their fields.

Julie's findings are summarized here:

- $n(W) = 160$              $n(E \cap F) = 90$
- $n(F) = 140$              $n(W \cap F \cap E) = 80$
- $n(E) = 130$              $n(E') = 95$
- $n(W \cap F) = 95$          $n[(W \cup F \cup E)'] = 10.$

Find the total number of students interviewed.

Solutions:



- 22. (a) 0 (b) 4 (c) 3
- (d) 0 (e) 6    23. (a) 51 (b) 31 (c) 18 (d) 15 (e) 33 (f) 23    24. (a) 37 (b) 22 (c) 50 (d) 11
- (e) 25 (f) 11    25. 225